

FIBER REINFORCED CEMENT AND CONCRETE COMPOSITES

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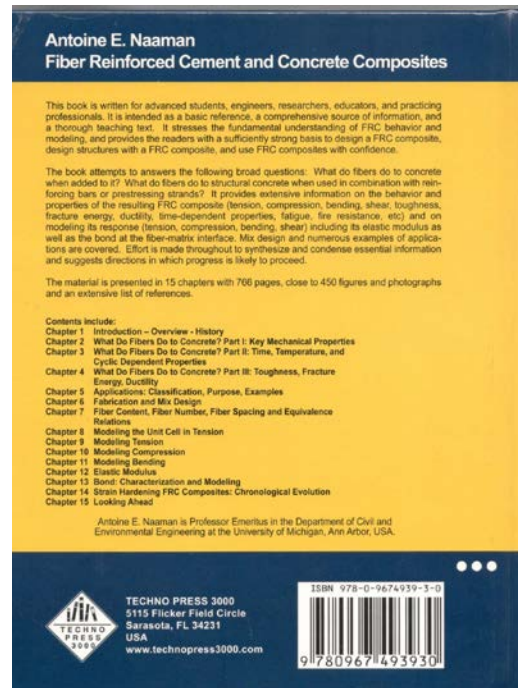
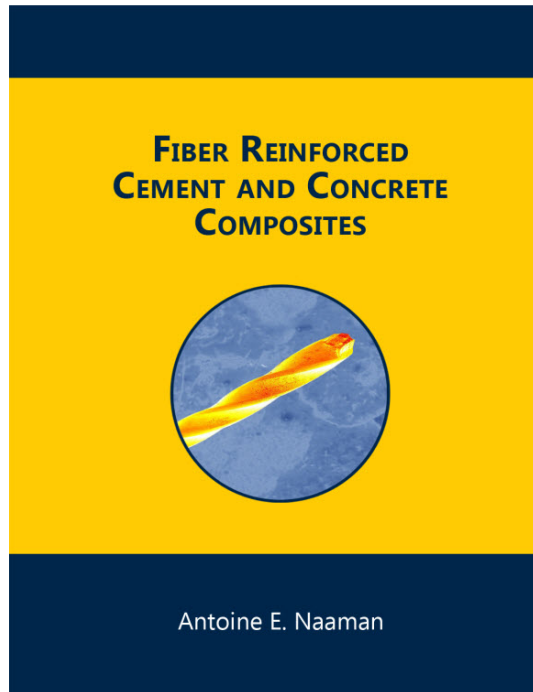
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Hardcover; 765 pages; 7.25x9.5 in.

ISBN 978- 0-9674939-3-0 LCCN 2017916342 - 2018

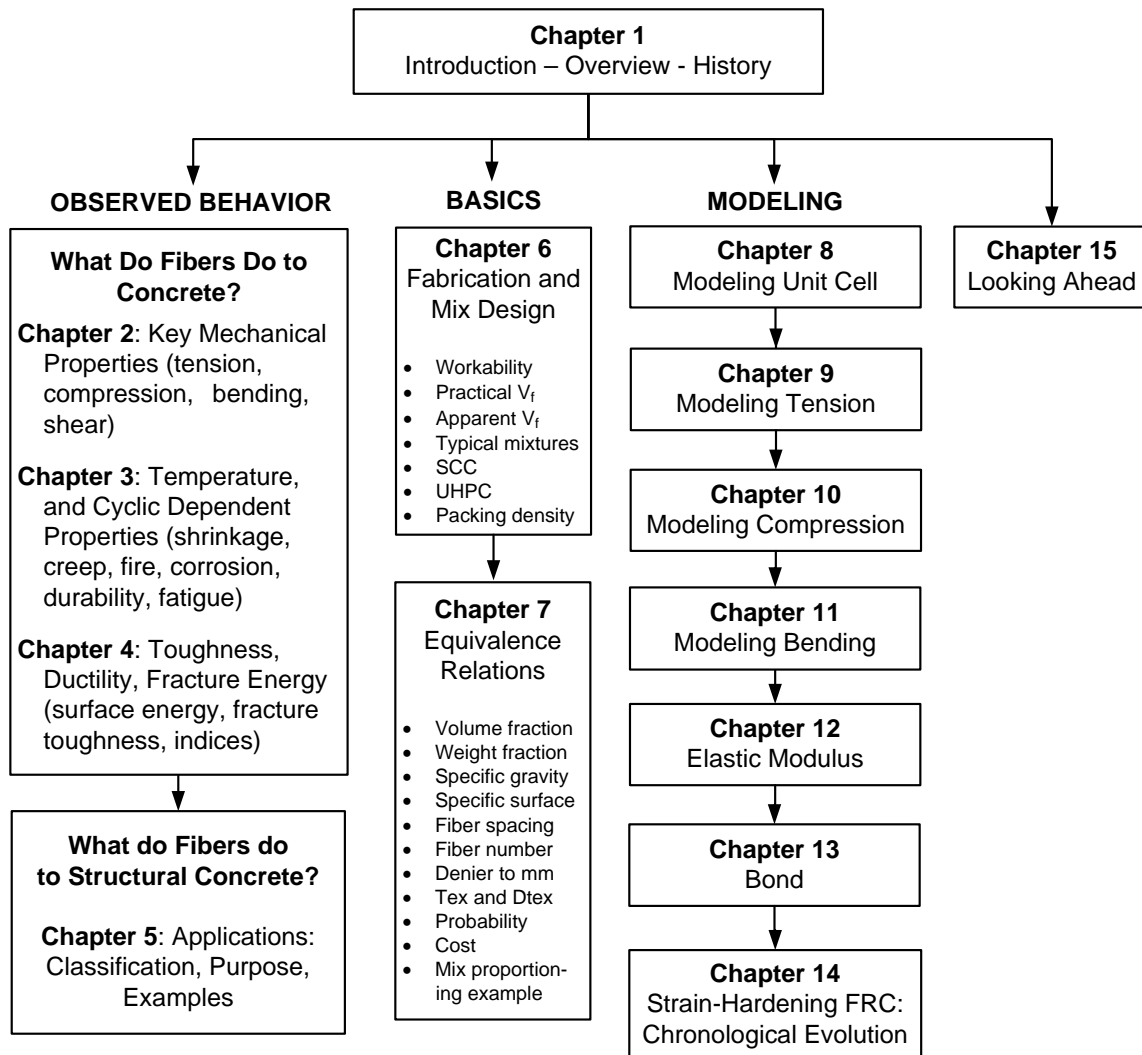


This book is written for advanced students, engineers, researchers, educators, and practicing professionals. It is intended as a basic reference, a comprehensive source of information, and a thorough teaching text. It stresses the fundamental understanding of FRC behavior and modeling, and provides the readers with a sufficiently strong basis to design an FRC composite, design structures with an FRC composite, and use FRC composites with confidence.

The book is organized into 15 chapters (see below), which can be assembled according to their intended function. Chapter 1 presents a general background and overview of historical evolution of fibers and FRC composites. The following three chapters address the question: What do fibers do to concrete as a material? They offer extensive information on the behavior and properties of the resulting composite (tension, compression, bending, shear, toughness, fracture energy, ductility, time-dependent properties, fatigue, fire resistance, etc). Chapter 5 describes different classes of applications, their purpose, and numerous examples to answer the question: What do fibers do to structural concrete? Chapter 6 deals with fabrication methods and mix design of FRC composites; potential challenges in manufacturing the composite, including proper fiber dispersion and mix workability, are addressed with suggested solutions. Chapter 7 develops the fundamental relations related to fiber and matrix volumes or weights and their fractions, fiber count, fiber spacing, specific surface, equivalence relations, and the like.

Chapters 8 to 14 focus specifically on analytical modeling (tension, compression, bending, shear, elastic modulus, bond, stress-strain relationships, etc), and Chapter 15 offers a perspective looking at the challenges ahead. This book will surely help the reader find rational answers and solutions to the many questions that arise when dealing with fiber reinforced concrete.

Whenever possible, a consistent notation and widely accepted symbols are adopted, and all symbols used in the text are defined and summarized in Appendix A. The book utilizes primarily the International System of Units (SI); however, all important tables, figures, and design information, are given in dual units (SI and US). Conversion factors from US to SI units and equivalence relations for some dimensionally inconsistent equations are given in Appendix B. An extensive list of references is given at the end of each chapter.



CONTENTS

	Preface	xxiii
	Acknowledgments	xxv
Chapter 1	Introduction: Overview - History	1
1.1	Composites and Fiber Reinforced Composites	1
1.1.1	Fiber Reinforced Cement and Concrete (FRC) Composites	2
1.2	FRC Composites: Historical Brief	4
1.3	Development of Fibers for Concrete	5
1.3.1	Pioneering Developments	6
1.3.2	Modern Developments	10
1.3.3	Why the Initial Lack of Success	11
1.4	Classification and Characterization of Fibers for Cement and Concrete Matrices	12
1.4.1	Suggested Characterization for Evaluation of Mechanical Performance	13
1.4.2	Classification by Material Type	15
1.4.3	Typical Properties of fibers for Use in Concrete	16
1.4.4	Macro and Micro-Fibers	19
1.4.5	Fiber Mats	21
1.5	Common Ranges of Fiber Geometric Parameters	22
1.5.1	Typical Fiber Reinforcing Parameters and Characteristics Important for Design	23
1.6	Typical Range of V_f and Mixing Limitations	23
1.7	Fiber-Matrix Reinforcing Effectiveness: Basic Approach	25
1.8	Simplified Mechanics for Initial Design	27
1.9	Pros and Cons of Fibers Used in Concrete	28
1.10	Applications: Types and Level of Usage	31
1.11	Cost Considerations: Fiber Volume Fraction versus Weight Fraction	33
1.11.1	Background	33
1.11.2	Density Ratio and Effect on Cost	34
1.12	FRC versus other Fiber Reinforced Composites: Fundamental Difference	35
1.13	FRC Composites: Overall Design and Challenges	37
1.14	The Continuous Path to Higher Performance	38
1.14.1	Whiskers and Nano-Fibers	38
1.15	Introduction to Other Chapters and Conclusion	39
1.16	References	41
	Books, Symposia, Web Sites, US Patents, References Cited	
Chapter 2	What Do Fibers Do To Concrete: Part I Key Mechanical Properties	47
2.1	Introduction	47

2.2	Understanding the Benefits, Drawbacks and Limitations of Fiber Reinforcement	49
2.2.1	Changes at the Material Level: Fresh State	50
2.2.2	Changes at the Material Level: Hardening State	52
2.2.3	Changes at the Material Level: Hardened State	52
2.2.3.1	Transition Zone	53
2.2.4	Changes at the Structural Level	53
2.2.5	Matrix Compatibility and Optimization	53
2.2.6	Trade-Offs, Limitations and Caution	54
2.3	What Do Fibers Do to Concrete in Tension?	55
2.3.1	Brittle and Ductile Materials	55
2.3.2	Typical Failure Modes in Tension	56
2.3.3	General Classification: Strain-Softening and Strain-Hardening Composites	57
2.3.4	Stress at First Cracking or First Percolation Cracking	61
2.3.5	Direct Tension Testing	62
2.3.6	Typical Stress-Strain or Stress-Elongation Curves in Direct Tension	65
2.3.7	Typical Stress versus Crack Opening Curves	70
2.3.8	Remarks: Reliability, Variability, Size Effect, Rate Effect	73
2.4	What Do Fibers Do to Concrete In Compression?	75
2.4.1	Reference Behavior of Plain Concrete in Compression	75
2.4.2	How Do Fibers Affect the Failure Mode of Concrete in Compression?	77
2.4.3	Stress-Strain Curve of FRC in Compression	80
2.4.4	Summary: Influence of Fibers on the Stress-Strain Response of Concrete in Compression	88
2.5	What Do Fibers Do to the Response of Concrete in Bending?	89
2.5.1	Typical Bending Response	89
2.5.2	Examples of Load-Deflection Curves as Influenced by Various Parameters	91
2.5.3	Correlation Between Bending and Tensile Response	98
2.6	What Do Fibers Do to the Response of Concrete in Shear?	100
2.6.1	Shear Tests	100
2.6.2	Shear Stress versus Strain Behavior	103
2.6.3	Behavior of SIFCON under Monotonic Shear Loading	105
2.6.4	Behavior of SIFCON under Cyclic Shear Loading	110
2.6.5	Analytical Predictions of Shear Strength of FRC	112
2.6.6	ACI Minimum Shear Reinforcement Comparison	115
2.7	SIFCON: Properties in a Nutshell	115
2.7.1	Special Applications of SIFCON	116
2.8	Early Age Compression Properties	117
2.9	Summary: Definitions and Terminology	119
2.10	Concluding Remarks	120
2.11	References	120

Chapter 3 What Do Fibers Do to Concrete? Part II: Time, Temperature, and Cyclic Dependent Properties 127

3.1	Introduction	127
3.2	Plastic Shrinkage Cracking Control	128
3.3	Shrinkage Strain	133
3.3.1	General Observations	134

	3.3.2 Free Shrinkage	135
	3.3.3 Restrained Shrinkage	136
3.4	Creep	138
	3.4.1 Creep Under Compression Loading	139
	3.4.2 Creep Under Tensile Loading	140
3.5	Fire Resistance	141
	3.5.1 General Observations	141
	3.5.2 Material Level	142
	3.5.3 Structural Level	144
	3.5.4 Special Case of PP Fiber and Similar Fiber Materials	146
	3.5.5 Use of Temperature for Property Enhancement and Modification	146
3.6	Corrosion Resistance, Deterioration, Degradation	147
	3.6.1 Corrosion: Background and Consequences	148
	3.6.2 Corrosion of Fiber Reinforced Concrete	150
	3.6.2.1 Experimental Variables	150
	3.6.2.2 Mechanical Tests	151
	3.6.2.3 Main Conclusions from Corrosion Testing	151
	of FRC	
	3.6.2.4 Tensile Post-Cracking Response as Affected by Corrosion or Other Degradation	155
3.7	Durability Issues and Freeze-Thaw Resistance	156
	3.7.1 Background on Freeze-Thaw Testing	156
	3.7.2 Relevant Observations for Plain Concrete	157
	3.7.3 Freeze-Thaw Resistance of Fiber Reinforced Concrete	158
	3.7.4 The Need for New Test Standards Suitable for FRC Composites	159
3.8	Fatigue	160
	3.8.1 Reference Fatigue for Plain Concrete in Compression	160
	3.8.2 Fatigue of Fiber Reinforced Concrete: Special Care and Precautions	161
	3.8.3 Fatigue of FRC in Bending with 2% Hooked Steel Fibers by Volume	164
	3.8.3.1 Key Experimental Observations	167
	3.8.3.2 S-N Relation	168
3.9	High Amplitude Cyclic Loading	169
3.10	Concluding Remarks	171
3.11	References	172

Chapter 4 What Do Fibers Do To Concrete? Part III: Toughness, Fracture Energy, Ductility 179

4.1	Introduction	179
	4.1.1 Load Test versus Deformation Test	180
	4.1.2 Special Notation for this Chapter	182
4.2	Toughness and Toughness Index	182
	4.2.1 ASTM C1018 Standard Test for Flexural Toughness	183
	4.2.1.1 Conceptual Basis	183
	4.2.1.2 Toughness Index Computation	184
	4.2.1.3 Remarks on First Cracking Point	186
	4.2.1.4 Some Issues with Toughness Indices	187
	4.2.2 ASTM C1609M Standard Test for Flexural Performance	188
	4.2.2.1 Remarks	191

	4.2.3 JCI-SF4 Methods of Tests of Flexural Strength and Flexural Toughness of Steel FRC	192
	4.2.4 RILEM TC 162-TDF Test and Design Methods for Steel FRC	193
	4.2.5 European Standard EN 14651	196
4.3	Conceptual Definition of Toughness and Energy Absorption Capacity	197
	4.3.1 Total Energy to Failure in Tension: Qualitative Observations	197
4.4	Basic Fracture Mechanics as Applied to FRC Composites	199
	4.4.1 Fracture Toughness and Stress Intensity Factor: Stress Based Criterion	201
	4.4.2 Fracture Energy and Toughness: Energy Based Criterion	204
	4.4.3 Correlation Between Fracture Energy and Fracture Toughness	205
	4.4.4 Non-Linear Materials	206
	4.4.5 Typical Experimental Tests to Obtain G_{Ic} and K_{Ic}	207
	4.4.5.1 Notched Tensile Prisms	208
	4.4.5.2 Compliance Method	209
	4.4.6 Multiple Criteria for Fracture	211
4.5	Fracture Processes in Fiber Reinforced Cement Composites: Experimental Observations	211
	4.5.1 Background	211
	4.5.2 Crack Model and R-Curve Behavior	212
	4.5.3 Examples of Numerical Results for Fracture Energy	217
	4.5.3.1 Fracture Energy for Strain-Hardening FRC Composites	218
4.6	Surface Energy of FRC Composites	220
	4.6.1 All Fibers Fail at Failure of the Matrix at the Cracked Section	220
	4.6.2 Some Fibers Fail and All Remaining Fibers Pull Out from the Cracked Section	221
	4.6.3 All Fibers Pull-Out from the Cracked Section	221
	4.6.4 Surface Energy of Pull-Out	222
	4.6.4.1 Theoretical Prediction from a Tension Test	222
	4.6.4.2 Experimental Results from Tension Tests	225
	4.6.4.3 Experimental Results from Bending Tests	227
4.7	Ductility and Ductility Index for Deflection-Hardening FRC Beams	227
	4.7.1 Conventional Definition of Ductility	227
	4.7.2 Ductility Index Applied to Deflection-Hardening FRC Composites	229
	4.7.3 Energy Based Ductility Index for Deflection-Hardening FRC Composites	230
	4.7.4 Example of Application	233
	4.7.5 Remarks on Ductility Index	235
4.8	Concluding Remarks	236
4.9	References	236
Chapter 5	Applications: Classification, Purpose, Examples	243
5.1	Introduction: Applications Classification	243

5.2	Applications by Fiber Type and Limitations	248
5.3	Design Trade-Offs for a Particular Application	251
5.4	Using SIFCON in Over-Reinforced Concrete Beams	252
5.5	Using Fibers in Concrete Beams Reinforced with Prestressing Strands	255
5.6	Using Fibers in Bridge Decks with Reduced Reinforcement	258
5.7	Effect of Fibers on Punching Shear Behavior of RC Decks, Slabs, Foundations	264
	5.7.1 Conclusions from Study on Punching Shear of HPFRCC Slabs	271
5.8	Moment-Rotation Response of Beams with Fibers, Reinforcing Bars, or Prestressing Strands	272
	5.8.1 Conclusions from Study on Moment-Rotation Capacity of HPFRCC Beams	275
5.9	FRC Seismic Resistant Joints, Connections, Coupling Beams, etc...	276
	5.9.1 Typical Examples	276
	5.9.2 Typical Experimental Tests for Joints and Connections	278
	5.9.3 Typical Experimental Tests for Coupling Beams	281
	5.9.4 Remarks on Material Design and Joint Design	283
	5.9.5 General Conclusion on Energy Absorption Capacity	284
	5.9.6 Enhancement of Material Toughness	285
	5.9.7 Resilience, Robustness and Damage Tolerance	285
5.10	Bond of Deformed Reinforcing Bars	286
	5.10.1 Bond Stress-Slip Relationship	286
	5.10.2 Pull-Out Test Specimen and Special Testing Fixture	287
	5.10.3 Bond of Strain-Hardening FRC and SIFCON	287
	5.10.4 Effect of Fiber Type on Bond Stress versus Slip Response	292
5.11	Bond of Prestressing Strands	298
	5.11.1 Conclusions from Tests on Prestressing Strands	299
	5.11.2 General Conclusions from Tests on Strands	304
5.12	Bond of Reinforcing Bars from Beam Tests	305
	5.12.1 Conclusions from Bond of Bars Embedded in SIFCON Beams	307
5.13	Improving Shear Resistance of Structural Concrete with Fibers	311
5.14	Summary: What Do Fibers Do to Structural Concrete?	312
5.15	Desirable Properties of Repair Materials	313
5.16	Concluding Remarks	313
5.17	References	314
Chapter 6	Fabrication and Mix Design	321
6.1	Introduction	321
	6.1.1 Cementitious Matrices	322
6.2	Manufacturing – Fabrication - Production	323
	6.2.1 Premixing	324
	6.2.1.1 Mixers	324
	6.2.1.2 Steps for Pre-Mixing	325
	6.2.1.3 Common Problems with Pre-Mixing	325
	6.2.1.4 Methods to Facilitate Fiber Addition to the Mixer	326
	6.2.1.5 Lay-Up Placement Method	327
	6.2.2 Shotcrete	327
	6.2.3 Spray-Up Process	328

6.2.4	Hatschek and Similar Machines	328
6.2.5	Extrusion	328
6.2.6	Slurry Infiltrated Fiber Concrete - SIFCON	329
6.3	Mix Design	329
6.4	What Do Fibers Do to Fresh Concrete: Key Parameters and Design Trade-Offs	330
6.4.1	Workability	330
6.4.2	Aggregate to Fiber Relative Size and Matrix Type	332
6.4.3	Practical Ranges of Fiber Content	334
6.4.4	Apparent versus Actual Volume Fraction	337
6.4.5	Understanding the Limitation on Maximum V_f or $V_f L/d$	338
6.4.6	Fiber Efficiency	341
6.4.7	Cost Considerations	341
6.4.8	Overall Design Parameters and Trade-offs	341
6.5	Examples of Mixtures for Steel Fiber Reinforced Concrete	342
6.6	Example Study on Self-Consolidating Strain-Hardening FRC Composites	346
6.6.1	Mixing Procedure for Self Consolidating FRC Mixtures	348
6.6.2	Remarks	349
6.7	Ultra-High Performance Fiber Reinforced Concrete: UHP-FRC	350
6.7.1	Background and Definition	350
6.7.2	Mix Design: Example of Experimental Study	352
6.7.3	Materials Selection	352
6.7.4	Mixing Procedure	354
6.7.5	Typical UHPC and UHP-FRC Mixtures	354
6.8	Particle Packing Considerations and Increasing Matrix Packing Density	356
6.9	Optimized Non-Proprietary UHP-FRC Mixtures	362
6.10	Precautions for Successful FRC Mixtures	364
6.11	Concluding Remarks	366
6.12	References	366

Chapter 7 Fiber Content, Fiber Number, Fiber Spacing and Equivalence Relations 371

7.1	Volume, Mass, Weight and their Fractions	371
7.1.1	Definitions, Notation, and Basic Relationships	372
7.1.2	Examples	376
7.1.3	Typical Examples of Fiber Weight (or Mass) for Normal Weight Concrete	378
7.2	Number of Fibers	378
7.2.1	Circular and Substantially Circular Fibers	378
7.2.2	Non-Circular Fibers	379
7.2.3	Examples: Number of Fibers	380
7.2.4	Typical Number of Fibers for $V_f = 1\%$	382
7.3	Average Fiber Spacing	383
7.3.1	Average Spacing of Fiber Intersections of a Plane	383
7.3.1.1	Circular Fibers	383
7.3.1.2	Non-Circular Fibers	385
7.3.2	Average Spacing of Fiber Centroids in Space	385
7.3.3	Examples: Fiber Spacing	386
7.3.4	Typical Fiber Spacing for $V_f = 1\%$	386
7.4	Specific Surface of Fibers	387

7.4.1	Circular or Primarily Circular Fiber Cross Section	388
7.4.2	Non-Circular Fibers	388
7.4.3	Examples of Specific Surface of Fiber Reinforcement	389
7.4.4	Example: Specific Surface Comparison between Reinforced Concrete and Fiber Reinforced Concrete	390
7.5	Equivalence Relations	391
7.5.1	Denier to Diameter	391
7.5.2	Examples: Denier to Diameter	392
7.5.3	Tensile Strength Conversion: Pounds per Denier to Pounds per Square Inch (psi) or to Newton/mm ² (MPa)	393
7.5.4	Examples: Tensile Strength Equivalence	393
7.5.5	Tex and Dtex	394
7.6	Probabilistic Analysis Related to N_v and N_s	394
7.6.1	Proof: Number of Fibers Crossing a Unit Area of Plane	396
7.6.2	Example: Random Number of Fiber Intersections	398
7.6.3	Influence of Mold Walls on the Value of α_2	399
7.7	Cost Considerations: Volume Fraction, Weight Fraction and Willingness-to-Pay Price	400
7.8	Example of Mix Proportioning by Weight or Volume	402
7.9	Concluding Remark	404
7.10	References	405

Chapter 8 Modeling the Unit Cell in Tension 407

8.1	Introduction	407
8.2	Nomenclature and Definitions	409
8.3	Assumptions for Modeling	410
8.4	Physical Model of Composite in Tension and Unit Cell	411
8.5	Stress Distribution Prior to Matrix Cracking in Unit Cell Model	414
8.5.1	Stresses in Fiber and Matrix within Transfer Distance	414
8.5.2	Distance at Which Equal Strains Occur: Transfer Distance x_0	419
8.5.3	Stresses in Matrix and Fiber at and Beyond the Transfer Distance x_0 Prior to Matrix Cracking	422
8.5.3.1	Trapezoidal Stress Profile	422
8.5.3.2	Triangular Stress Profile	423
8.5.4	External Tensile Load Leading to First Matrix Cracking	423
8.5.5	Particular Value of Transfer Distance at Onset of Matrix Cracking	424
8.5.6	Stresses in Matrix and Fiber for $T = T_{cr}$ Just Prior to Matrix Cracking	425
8.5.7	Stresses in Matrix and Fiber Just After Cracking	426
8.6	Average Stress in the Fiber at Onset of Cracking	427
8.6.1	Partial Bond Activation at Fiber-Matrix Interface: Trapezoidal Stress Profile in Fiber	430
8.6.2	Full Bond Activation at Fiber-Matrix Interface: Triangular Stress Profile in Fiber	432
8.7	Examples: Stress Distribution in Unit Cell Model at Onset of Matrix Cracking	433
8.7.1	Steel Fiber Reinforced Concrete: Fiber Type 1	433
8.7.2	Steel Fiber Reinforced Concrete (UHPC): Fiber Type 2	436

	8.7.3 Steel Fiber Reinforced Concrete (UHPC): Fiber Type 3	437
	8.7.4 Slurry Infiltrated Fiber Concrete: SIFCON	437
	8.7.5 PVA Fiber Reinforced Cement Composite: Coarse Fiber	438
	8.7.6 PVA Fiber Reinforced Cement Composite: Fine Fiber	439
	8.7.7 PVA Fiber Reinforced Aerated Concrete	440
	8.7.8 Polypropylene Fiber Reinforced Cement Composite	440
	8.7.9 Carbon Fiber Reinforced Cement Composite	441
	8.7.10 Spectra Fiber Reinforced Cement Composite	442
	8.7.11 Asbestos Fiber Reinforced Cement Composite	443
	8.7.12 Unit Cell in Compression: Steel Fiber Reinforced Concrete	444
8.8	Practical Observations	444
	8.8.1 Stress Estimate in Overall Composite at Onset of Matrix Cracking	446
8.9	Concluding Remark	448
8.10	References	448
Chapter 9	Modeling Tension	451
9.1	Introduction	451
	9.1.1 Notation	452
9.2	Simplified Stress-Elongation Models and Assumptions	453
	9.2.1 Assumptions	455
	9.2.2 Design Strategy	456
9.3	Tensile Stress and Strain in Composite at First Percolation Cracking ($\sigma_{cc}, \varepsilon_{cc}$)	457
	9.3.1 Stress at First Cracking σ_{cc}	457
	9.3.2 Strain at First Cracking ε_{cc}	459
	9.3.3 Upper Bound Stress in Composite at Cracking of Matrix	460
	9.3.4 Proof Leading to Eq. (9.1)	460
9.4	Maximum Post-Cracking Stress (Strength) and Strain in Tension ($\sigma_{pc}, \varepsilon_{pc}$)	462
	9.4.1 Possible Cases for Maximum Post-Cracking Stress, σ_{pc}	462
	9.4.1.1 Composite Strength Assuming All Fibers Fail Simultaneously	463
	9.4.1.2 Composite Strength Assuming All Fibers Pull-Out Simultaneously	464
	9.4.2 Non-Dimensional Form for σ_{pc}	468
	9.4.3 Upper Bound Limit for σ_{pc}	468
	9.4.4 Derivations Leading to Eq. (9.18) for σ_{pc}	468
	9.4.4.1 Unidirectional Fibers	468
	9.4.4.2 Randomly Oriented Fibers	471
9.5	Strain at Maximum Post-Cracking Stress, ε_{pc}	472
9.6	General Pull-Out Response After σ_{pc} : Part III of Figure 9.3	474
	9.6.1 Kosa and Naaman - 1990	474
	9.6.2 Visalvanich and Naaman - 1983	475
	9.6.3 Gopalaratnam and Shah - 1985, 1987	476
	9.6.4 Li - 1992	476
9.7	Summary: Idealized Tensile Response for Modeling	477

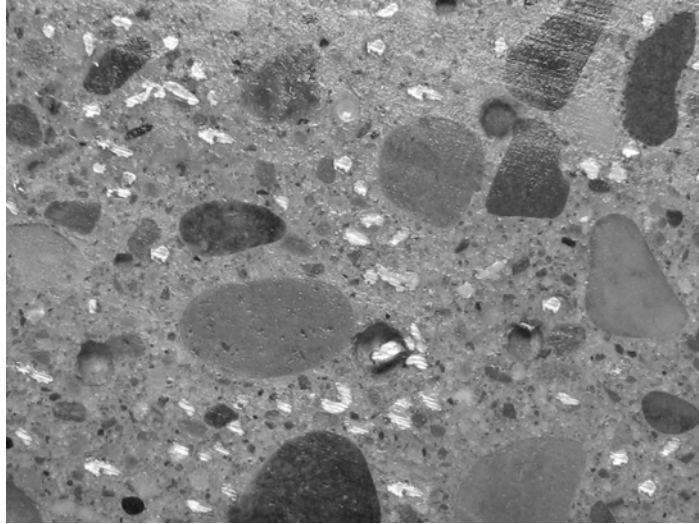
9.8	Critical Volume Fraction of Fibers to Achieve Strain-Hardening Behavior in Tension	478
	9.8.1 Graphical Illustration	480
9.9	Critical Volume Fraction of Fibers to Achieve Deflection-Hardening Behavior in Bending	481
	9.9.1 Example: Critical Volume Fraction of Fibers	483
	9.9.2 Practical Limitations	484
9.10	Numerical Examples of Cracking and Maximum Post-Cracking Tensile Stresses	484
	9.10.1 Steel Fiber Reinforced Mortar or Concrete	484
	9.10.2 Steel Fiber Reinforced Concrete – Fibers with High Bond Strength	485
	9.10.3 Slurry Infiltrated Fiber Concrete - SIFCON	486
	9.10.4 Ultra-High Performance Fiber Reinforced Concrete, UHP-FRC	487
	9.10.5 Carbon Fiber Reinforced Cement Composite	488
	9.10.6 PVA Fiber Reinforced Cement Composite, Coarse Fiber	489
	9.10.7 PVA Fiber Reinforced Cement Composite, Fine Fiber	490
	9.10.8 PVA Fiber Reinforced Aerated Concrete	491
	9.10.9 Polypropylene Fiber Reinforced Cement Composite	491
	9.10.10 Polypropylene Fiber Reinforced Lightweight Cement Composite	492
	9.10.11 Glass Fiber Reinforced Cement Board	493
	9.10.12 Fiber Reinforced Cement Composite with Spectra Fiber	494
	9.10.12 Steel Fiber Reinforced Mortar with Rectangular Fiber	495
9.11	Modeling a More Complete Tensile Stress-Elongation Curve	496
9.12	Desirable Characteristics of the Tensile Stress-Strain Curve for Structural Design	498
9.13	Fiber Effectiveness and Efficiency	499
	9.13.1 Fiber Reinforcing Index	499
	9.13.2 Fiber Intrinsic Efficiency Ratio	500
	9.13.3 Generation of Other Fiber Cross-Sectional Shapes	502
9.14	Concluding Remark	503
9.15	References	504
Chapter 10	Modeling Compression	511
10.1	Introduction	511
10.2	Parameters Influencing the Stress-Strain Response in Compression	514
	10.2.1 Experimental Observations	514
	10.2.2 Influence of Testing Variables	514
10.3	Stress-Strain Equations for Compression	515
	10.3.1 Notation	515
	10.3.2 Common Boundary Conditions	517
	10.3.3 Analytical Relationships: Primarily for Plain Concrete	519
	10.3.4 Recommended Analytical Equation for Plain Concrete	523
	10.3.5 Recommended Analytical Equations for Fiber Reinforced Concrete	525
	10.3.5.1 Exponential Equation for Inverted S-Curve:	527
	Descending Branch	

10.4	Example: Curve Fit for Fiber Reinforced Concrete	529
10.4.1	Ascending Branch: Parabola	530
10.4.2	Ascending Branch: Fractional Equation	531
10.4.3	Ascending Branch: Exponential Function	531
10.4.4	Descending Branch: Fractional Equation	532
10.4.5	Descending Branch: Exponential Passing by Two Prescribed Points	534
10.4.6	Descending Branch: Exponential with Zero Curvature at Inflection Point	535
10.4.7	Recommendations	536
10.5	Example: Curve Fit for Plain Concrete	536
10.5.1	Analytically Generated Stress-Strain Relation for: $f'_c = 69 \text{ MPa} = 10 \text{ ksi}$	536
10.5.2	Curve Fit of Experimental Curve for Plain Concrete	537
10.5.2.1	Ascending Branch	537
10.5.2.2	Descending Branch	538
10.6	Implementation in Nonlinear Analysis	539
10.6.1	Example: Resultant Location Using Parabolic Stress Block	542
10.7	FRC Under Cyclic and Random Amplitude Loading	544
10.8	Concluding Remarks	548
10.9	References	548
Chapter 11	Modeling Bending Response	551
11.1	Introduction - Background	551
11.1.1	Notation Used in This Chapter	552
11.1.2	Assumptions for Modeling	552
11.1.3	Elastic Moment and Plastic Moment	554
11.1.4	Plastic Material with Different Yield Strengths in Tension and compression	555
11.2	Bending Behavior of FRC Composites	557
11.2.1	Typical Load-Deflection Curves and Correlation with Tensile Response	557
11.2.2	Cracking Moment	558
11.2.3	Modulus of Rupture	559
11.2.4	Section Response at First Cracking: Hinge Behavior	559
11.2.5	Definition of Average Post-Cracking Stress for Design	560
11.3	Nominal Bending Resistance, M_n	562
11.3.1	Compression and Tensile Stress Profiles	562
11.3.2	ACI Rectangular Stress Block and Uniform Stress Distribution in Tension	564
11.3.2.1	Examples	566
11.3.3	Triangular Stress Block in Compression and Uniform Stress Distribution in Tension	570
11.3.3.1	Example	571
11.3.4	Parabolic Stress Block in Compression and Uniform Stress Distribution in Tension	572
11.3.4.1	Example	574
11.3.5	ACI Stress Block in Compression and Triangular Stress Distribution in Tension	575
11.3.5.1	Example	576
11.4	General Observations and Discussion	577
11.4.1	Simplified Results Assuming: ACI Rectangular	578

	Stress Block and Uniform Tensile Stress	
	Distribution	
	11.4.2 Condition for Deflection Hardening for a Rectangular Section	581
11.5	General Condition for Deflection-Hardening FRC Composites	582
11.6	Influence of Crack Opening on Average Tensile Stress Resistance	583
11.7	Computation of Deflections in FRC Beams	587
11.8	Concluding Remark	588
11.9	References	589
Chapter 12	Elastic Modulus	593
12.1	Background on Modulus of Elasticity	593
	12.1.1 Shear and Bulk Modulus	596
12.2	Prediction Models for the Elastic Modulus of Composites	596
12.3	Fibers with Axis Parallel to Loading Direction: Upper Bound Equal-Strain Solution (Voigt's Model)	598
	12.3.1 Proof of Voigt's Model	599
	12.3.2 Examples of Upper-Bound Modulus: Unidirectional Fibers Parallel to Loading Direction	600
12.4	Fibers with Axis Normal to Loading Direction: Lower-Bound Equal-Stress Solution (Reuss's Model)	602
	12.4.1 Proof of Reuss's Solution	603
	12.4.2 Examples of Lower Bound Modulus: Fibers Normal to Loading Direction	605
	12.4.3 Comparison Summary	606
12.5	Other Models for Unidirectional Discontinuous Fibers	606
	12.5.1 Cox Model (1952) for Unidirectional Short Fibers of Length L	606
	12.5.2 Example Using Cox's Prediction Equation	607
	12.5.3 Halpin and Tsai's Model (1969)	608
	12.5.4 Example Using Halpin and Tsai's Prediction Equation	609
12.6	Models for Randomly Oriented Discontinuous Fibers	609
	12.6.1 Cox's Solution for a Two-Phase Model	613
	12.6.2 Halpin and Tsai's Solution for a Two-Phase Model	613
	12.6.3 Najm and Naaman's Solution	614
	12.6.4 Alwan and Naaman's Solution for a Pseudo Three-Phase Model	615
12.7	Practical Recommendation for Predicting the Elastic Modulus of FRC Composites with Randomly Oriented and Distributed Fibers	617
12.8	Measurements of Elastic Modulus	620
12.9	Concluding Remarks	621
12.10	References	622
Chapter 13	Bond: Characterization and Modeling	625
13.1	Introduction	625
13.2	Bond Components	627
	13.2.1 Adhesive Bond: Chemical and Physical	627
	13.2.2 Frictional Bond	628
	13.2.3 Mechanical Bond: Anchorage and Clamping	630
	13.2.4 Fiber to Fiber Interlock	630
13.3	Basic Pull-Out and Pull-Through Tests	631

13.3.1	Preferred Behavior of a Fiber under Pull-Out	631
13.3.2	Typical Pull-Out and Pull-Through Test Set-ups	633
13.3.3	Basic Calculations for Bond or Shear Stress	635
13.3.4	Average Bond Stress and Equivalent Bond Stress	636
13.3.5	Definition of Slip	638
13.4	Typical Pull-Out Response and Pull-Out Mechanism of Straight Smooth Steel Fibers	638
13.5	Typical Pull-Out Response and Pull-Out Mechanism of Hooked-End Steel Fiber	641
13.6	Typical Pull-Out Response and Pull-Out Mechanism of Indented and Crimped Steel Fibers	644
13.7	Typical Pull-Out Response and Pull-Out Mechanism of Twisted Polygonal Steel Fiber	646
13.7.1	Observations with Twisted Fibers	653
13.8	Isolating the Mechanical Component of Bond	654
13.9	Pull-Out with UHPC	655
13.10	General Observations on the Pull-Out Response of Steel Fibers and Comparisons	657
13.11	Early Age Bond	659
13.12	Loading Rate Effect on Pull-Out	661
13.12.1	Pull-Out under Impact Loading	663
13.13	Pull-Out of Aligned and Oriented Steel Fiber and Influence of Number of Fibers Pulling Out	664
13.14	Analytical Modeling: Pull-Out Response and Bond Shear Stress versus Slip Relationship of Smooth Fibers	669
13.14.1	Simplified Assumptions for Modeling	669
13.14.2	Notation and Basic Equations	671
13.14.3	Solution of the Primal Problem: Prediction of the Entire Pull-Out Load versus Slip Response	673
13.14.4	Solution of the Dual Problem: Bond Shear Stress versus Slip Relationship	678
13.15	Slip Hardening Bond Shear Stress versus Slip Response	680
13.16	Analytical Modeling of Deformed Steel Fiber Pull-Out Behavior	681
13.16.1	Model for Corrugated and Crimped Steel Fiber	681
13.16.2	Model for Hooked-Ends Steel Fiber	682
13.16.3	Model for Fiber with End Anchorage	683
13.16.4	Model for Twisted Steel Fiber	683
13.17	Considerations for Structural Applications	686
13.17.1	Comparison of Typical Slip at Peak Pull-Out Load	686
13.17.2	Correlation with Crack Width in Structural Concrete Members	687
13.18	Pull-Out Work and Pull-Out Energy	689
13.19	Concluding Remarks	690
13.20	References	691
Chapter 14	Strain Hardening FRC Composites: Chronological Evolution	697
14.1	Introduction	697
14.2	Historical Background	698
14.3	Evolution of Nomenclature to Describe Strain Hardening Behavior	705
14.4	Evolution Summary	706
14.5	Conditions for Strain-Hardening Behavior: Fracture	707

14.6	Mechanics Approach versus Composite Mechanics Approach Similarities and Differences in the Known Conditions for Strain-Hardening Behavior	712
14.7	Converging Studies	714
14.8	Correlation Between Tensile Stress-Displacement Response and Tensile Stress-Strain Response	716
14.9	Concluding Remarks	716
14.10	References	718
Chapter 15	Looking Ahead	723
15.1	Introduction	723
15.2	Hybrid Fiber Reinforcement	724
15.3	Very Lightweight and Cellular Fiber Reinforced Concrete	726
15.3.1	Main Observations	727
15.4	Effect of Fibers on Ferrocement and Thin Cement Composites	730
15.5	Effect of Conventional Reinforcement on Tensile Response of FRC	732
15.5.1	Observations and Conclusions	733
15.6	The Need for Slip-Hardening Bond and Limitations	734
15.7	Self-Stressing FRC Composites	735
15.8	New and Improved Materials	736
15.9	Size Effects, Loading Rate Effects, Tension Stiffening	737
15.10	Fiber Consumption Estimates	738
15.10.1	Potential Growth	738
15.11	Looking Ahead	739
15.12	References	741
Appendix A	List of Symbols	745
Appendix B	Unit Conversions	752
INDEX	INDEX	755
	Author	764



Typical sections of steel fiber reinforced concrete (top) and mortar (bottom).