

# FIBER REINFORCED CEMENT AND CONCRETE COMPOSITES

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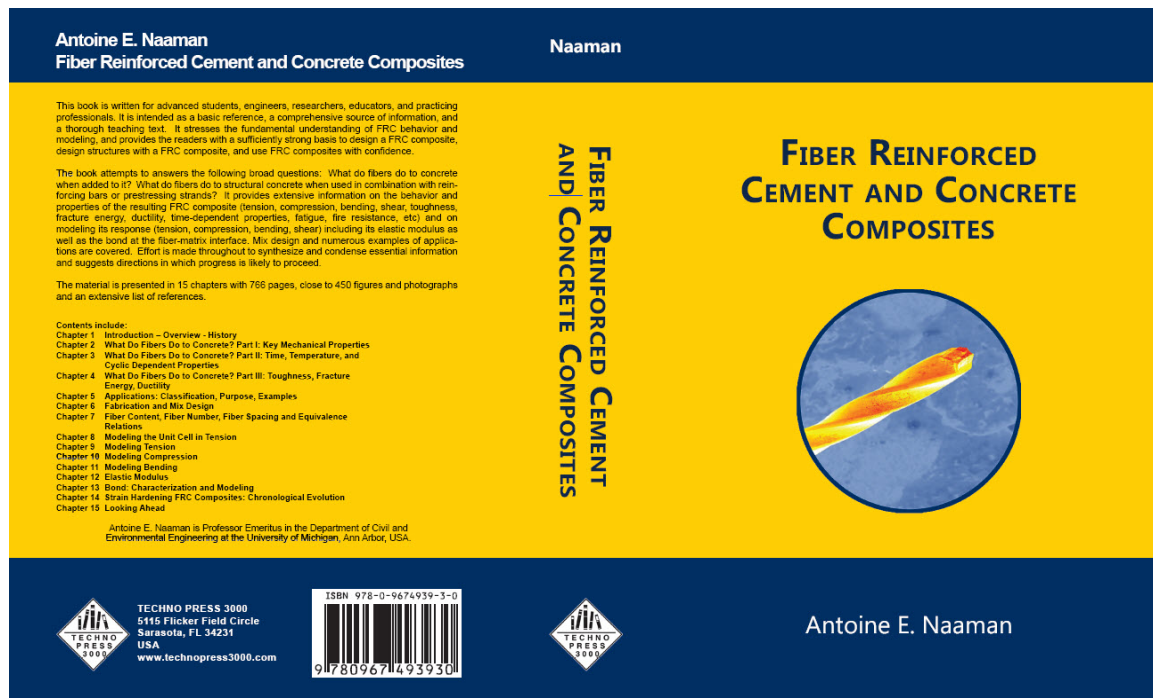
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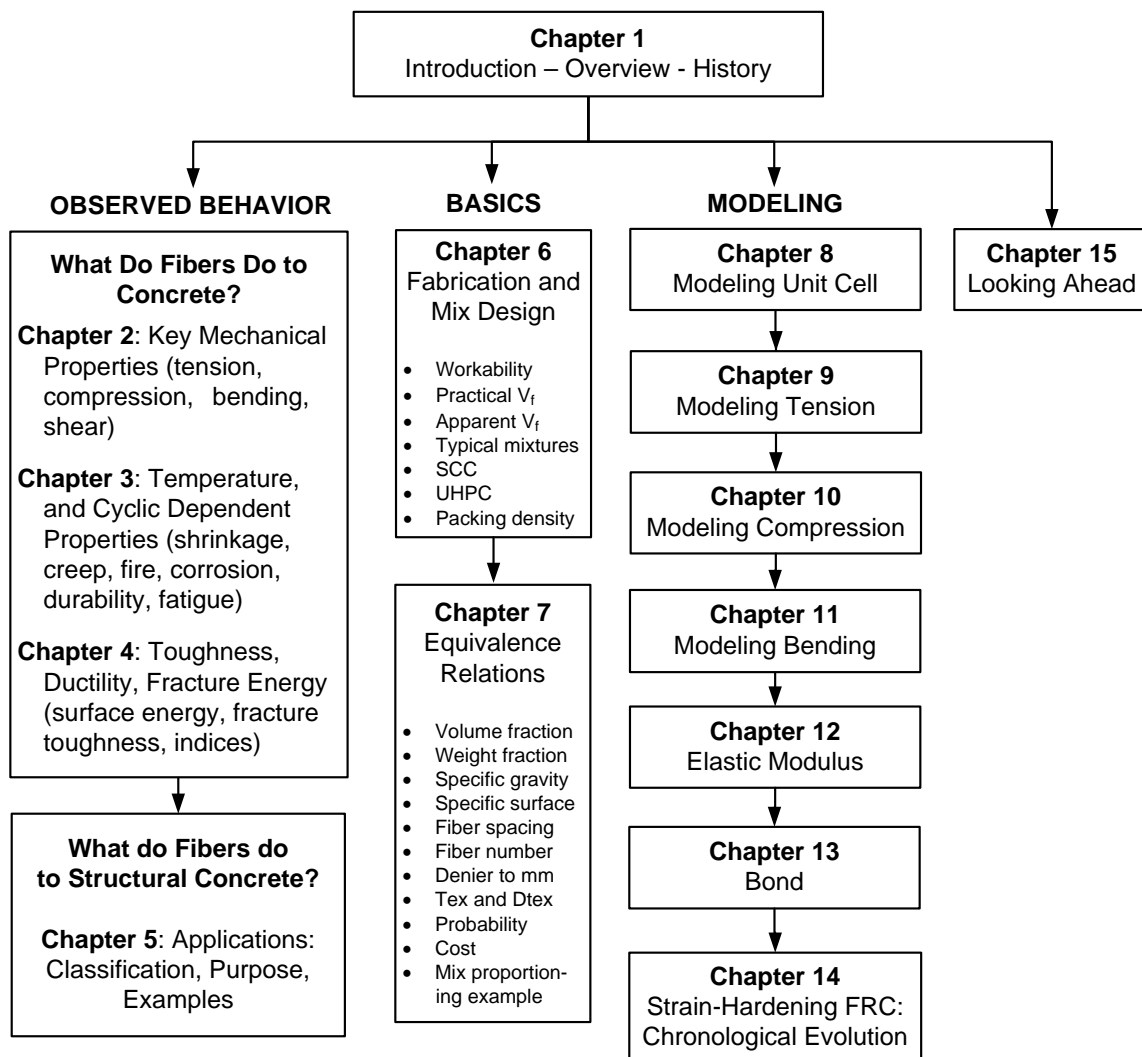


This book is written for advanced students, engineers, researchers, educators, and practicing professionals. It is intended as a basic reference, a comprehensive source of information, and a thorough teaching text. It stresses the fundamental understanding of FRC behavior and modeling, and provides the readers with a sufficiently strong basis to design an FRC composite, design structures with an FRC composite, and use FRC composites with confidence.

The book is organized into 15 chapters (see below), which can be assembled according to their intended function. Chapter 1 presents a general background and overview of historical evolution of fibers and FRC composites. The following three chapters address the question: What do fibers do to concrete as a material? They offer extensive information on the behavior and properties of the resulting composite (tension, compression, bending, shear, toughness, fracture energy, ductility, time-dependent properties, fatigue, fire resistance, etc). Chapter 5 describes different classes of applications, their purpose, and numerous examples to answer the question: What do fibers do to structural concrete? Chapter 6 deals with fabrication methods and mix design of FRC composites; potential challenges in manufacturing the composite, including proper fiber dispersion and mix workability, are addressed with suggested solutions. Chapter 7 develops the fundamental relations related to fiber and matrix volumes or weights and their

fractions, fiber count, fiber spacing, specific surface, equivalence relations, and the like. Chapters 8 to 14 focus specifically on analytical modeling (tension, compression, bending, shear, elastic modulus, bond, stress-strain relationships, etc), and Chapter 15 offers a perspective looking at the challenges ahead. This book will surely help the reader find rational answers and solutions to the many questions that arise when dealing with fiber reinforced concrete.

Whenever possible, a consistent notation and widely accepted symbols are adopted, and all symbols used in the text are defined and summarized in Appendix A. The book utilizes primarily the International System of Units (SI); however, all important tables, figures, and design information, are given in dual units (SI and US). Conversion factors from US to SI units and equivalence relations for some dimensionally inconsistent equations are given in Appendix B. An extensive list of references is given at the end of each chapter.



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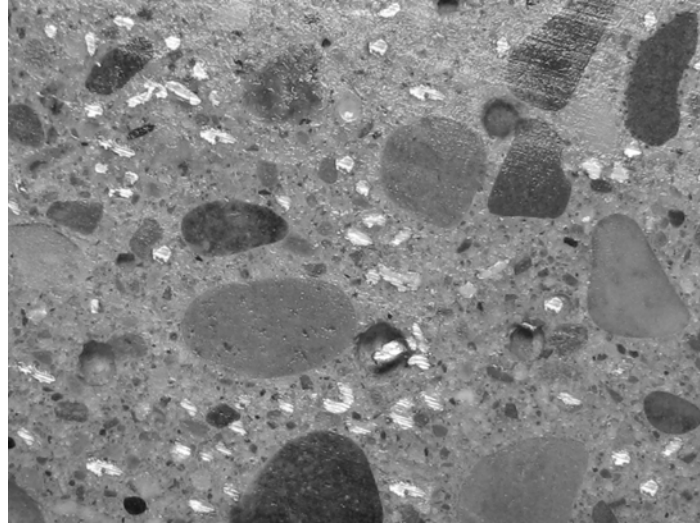
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**Typical sections of steel fiber reinforced concrete (top) and mortar (bottom).**